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Soft Time-Window Constraints in Crystal-Ice Distribution: A Cheapest-Insertion VRPTW Case Study with Non-Refrigerated Vehicles

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Abstract

Perishable product distribution requires efficient route planning because delivery duration, route sequence, and transportation conditions directly affect product quality, service reliability, and operational performance. This study evaluates hard and penalty-based soft time-window routing for crystal-ice distribution in a small-scale enterprise distribution system comprising one depot, 60 scheduled customers, and two non-refrigerated vehicles. A quantitative computational case-study approach was applied using the Vehicle Routing Problem with Time Windows (VRPTW) and the Cheapest Insertion Heuristic (CIH). Soft time-window performance was evaluated using lateness penalties of $\lambda = 0.1, 0.5, 1, 2$, and 5 km/min. The hard time-window solution produced a fleet distance of 86.87 km and zero lateness. At $\lambda = 0.1$, the soft model reduced the fleet distance to 85.29 km but generated 8.95 minutes of total lateness across two customers. Increasing the penalty to $\lambda = 0.5$ eliminated all lateness and produced the same 86.87-km route as the hard model. Compared with the observed company baseline, the selected zero-lateness route reduced fleet distance from 99.84 to 86.87 km (12.99%) and route exposure duration from approximately 1,200 to 1,043.59 minutes (13.03%). Model-estimated product loss decreased from an observed baseline of 580 kg to 507.95 kg, corresponding to an estimated reduction of 72.05 kg (12.42%). These findings show that soft time-window penalties can reveal the trade-off between marginal distance savings and service punctuality.

KEYWORDS

crystal ice distribution; perishable product; vehicle routing problem with time windows; cheapest insertion heuristic; soft time window; lateness penalty.

Introduction

Perishable product distribution has become an important issue in supply chain management because routing decisions, delivery duration, and transportation conditions directly affect product quality, service reliability, and operational performance (Andrianto et al., 2025; Y.-Y. Chen et al., 2023; Y. Liang et al., 2020; Nasrudin, 2026; Qin et al., 2019; Tirkolaei et al., 2020; Zulvia et al., 2020). This problem is particularly relevant in cold-chain and time-sensitive logistics, where prolonged distribution may accelerate product deterioration before products reach customers (Hsu et al., 2007; Wang et al., 2018). Reviews of perishable-product routing show that current models increasingly integrate time windows, deterioration, service quality, and operational constraints, while customer demand, vehicle capacity, service intervals, and route feasibility remain fundamental components of vehicle-routing decisions (X. Liu et al., 2023; Toth & Vigo, 2014; Utama et al., 2020).

Crystal ice distribution represents a specific form of time-sensitive delivery (Dewi, 2026; Thammasang & Arunyanart, 2024). Small and medium-sized crystal ice enterprises often operate vehicles without active refrigeration or chiller systems; consequently, product preservation depends largely on vehicle insulation, handling

conditions, route sequence, and the length of time the product remains inside the vehicle during distribution. Prolonged travel, waiting, and customer-service activities may therefore increase the exposure of crystal ice to melting-related loss.

For product-loss calibration in the investigated company, field observations were conducted over ten delivery days under regular distribution conditions. The observations reported average initial physical loads of approximately

Table 1. Selected Studies Relevant to Perishable-Product and Time-Window Routing

No	Author(s)	Year	Study Focus	Key Findings	Moderating Factors	Geographic Context
1	Hsu et al.	2007	Perishable product distribution	Product deterioration is influenced by transportation duration and delivery conditions. Longer delivery time increases the risk of product-quality degradation.	Product perishability; delivery duration	Numerical grid; no specific location
2	Wang et al.	2018	Product deterioration in distribution	Distribution performance should consider not only transportation cost but also deterioration risk and service reliability.	Product deterioration; delivery reliability	Computational study; no specific location
3	Figliozzi	2010	Soft time-window routing	Soft time-window structures allow early or late arrivals to be converted into penalty costs instead of making the route automatically infeasible.	Early arrival penalty; lateness penalty; service reliability	Solomon benchmark
4	Pillac et al.	2013	Vehicle Routing Problem with Time Windows	VRPTW extends classical VRP by incorporating customer service-time intervals, vehicle capacity, and route feasibility constraints.	Time windows; vehicle capacity; service feasibility	International review
5	Toth & Vigo	2014	Vehicle routing optimization	VRP provides a fundamental framework for determining vehicle routes under operational constraints such as customer demand and vehicle capacity.	Vehicle capacity; customer demand	General framework
6	Z. Liu et al.	2023	Perishable product routing with soft time windows	Soft time-window penalties can improve routing flexibility by balancing travel efficiency and lateness cost in perishable product distribution.	Lateness penalty; product perishability	Computational instances
7	He et al.	2024	Cold-chain routing and product preservation	Cold-chain distribution requires route planning that accounts for product-quality preservation and transportation-related deterioration risk.	Refrigeration condition; delivery duration	Solomon benchmark
8	Thammasang & Arunyanart	2024	Ice distribution routing	Ice distribution requires practical routing models that consider customer service requirements and quality-preservation concerns.	Customer retention; product preservation	Thailand
9	Y. Liu et al.	2025	Customer time satisfaction in cold-chain logistics	Expected, soft, and hard time-window structures can represent different levels of customer service satisfaction in routing problems.	Customer satisfaction; time-window violation	China; computational study
10	Rosenkrantz et al.	1977	Route-construction heuristics	Insertion-based heuristics construct routes by gradually inserting unserved nodes into existing routes based on additional travel cost.	Route sequence; insertion cost	Theoretical study
11	Y. Chen et al.	2023	Integrated cold-chain scheduling and route planning under demand surge	Integrated scheduling and route optimization reduced delivery time, distribution cost, and total mileage.	Demand surge; order batching; customer time windows; vehicle capacity	China

1,910.5 kg for Truck 1 and 2,055 kg for Truck 2, with average baseline distribution exposure durations of approximately 600 minutes per truck. Company operational records reported average melting-related product-loss proportions of 14.09% for Truck 1 and 15.13% for Truck 2. These field observations were used as the empirical baseline for estimating melting-related product loss under the optimized routing scenarios. Because crystal ice remained inside the non-refrigerated vehicles during travel, waiting, and customer-service activities, the full route exposure duration was considered relevant to product preservation rather than pure driving time alone.

Previous studies have demonstrated the relevance of Vehicle Routing Problem with Time Windows (VRPTW) approaches in perishable-product and cold-chain distribution (Agrawal et al., 2022; Babaei Tirkolaee et al., 2017; Y. Chen et al., 2023; Hsu et al., 2007; Ma et al., 2017; Song et al., 2020; Utama et al., 2020). Soft time-window formulations allow service-time violations to be represented through penalty mechanisms rather than automatically making a route infeasible (Figliozzi, 2010; Z. Liu et al., 2023). Other studies have emphasized the relationship between distribution duration, transportation conditions, and product preservation in perishable logistics (Deng et al., 2023; Estrada-Moreno et al., 2019; He et al., 2024; Wang et al., 2018; Wasil et al., 2025). In the context of ice distribution, Thammasang & Arunyanart (2024) also demonstrated the importance of routing decisions in balancing customer-service requirements and product-preservation concerns. Selected studies relevant to the methodological and operational context of the present research are summarized in Table 1.

Beyond conventional perishable-product routing, VRPTW research has also considered stochastic and time-dependent travel conditions (Bomboi et al., 2022), route consistency (Lespay & Suchan, 2021), robust data-driven routing (Zhang et al., 2021), dynamic routing (Pillac et al., 2013), and low-carbon cold-chain operations (Y. Liu et al., 2025; Marjan & Faisal, 2025). Optimization approaches have also been increasingly applied to last-mile logistics (Giuffrida et al., 2022), while recent perishable-product routing models have incorporated customer priorities and customized delivery time windows (X. Liang et al., 2024). These developments illustrate the broader range of operational considerations that can be incorporated into vehicle-routing decisions.

The studies summarized in Table 1 indicate that vehicle capacity, customer time windows, delivery duration, route sequence, and product preservation requirements are recurring considerations in perishable-product routing. Existing research also demonstrates that soft time-window penalties can provide routing flexibility by allowing lateness to be evaluated as part of the routing cost (Wang et al., 2018; Zhao et al., 2021). However, the operational settings and methodological emphases of previous studies differ from the present case. The current study examines a small scale crystal ice distribution operation using two non-refrigerated vehicles and combines scheduled customer demand, customer service time windows, field-observed melting conditions, road-network-based distance and travel time, and sensitivity analysis of the lateness-penalty parameter within a single case-based evaluation framework.

The present study does not propose a new VRPTW formulation or a new routing heuristic. Instead, its contribution is primarily empirical, contextual, and decision-oriented. An established penalty based soft time window mechanism is combined with the Cheapest Insertion Heuristic (CIH), while the lateness penalty parameter is evaluated through sensitivity analysis rather than selected subjectively. The analysis applies a minimum sufficient penalty rule, defined as the smallest tested penalty value that eliminates customer lateness while maintaining vehicle-capacity and

depot-return feasibility. In addition, routing performance is evaluated not only through travel distance and service punctuality but also through route exposure duration and a field calibrated estimate of melting-related product loss. The contribution should therefore be interpreted as a computationally verified case based application of established routing concepts rather than as the development of a new optimization method.

Accordingly, this study aims to evaluate hard and penalty based soft time window VRPTW routing using the Cheapest Insertion Heuristic for non-refrigerated crystal-ice distribution. The study specifically examines how different lateness penalty values influence the trade off between routing efficiency and customer-service punctuality and identifies the minimum penalty required to obtain a zero lateness solution while maintaining vehicle capacity and depot-return feasibility. The resulting routing alternatives are further evaluated in terms of travel distance, route exposure duration, lateness, operational feasibility, and estimated melting related product loss to provide a practical routing recommendation for the investigated distribution system.

Based on these objectives, the study addresses the following research questions: (1) How do hard and penalty based soft time-window VRPTW solutions differ in terms of travel distance, pure travel time, customer lateness, route exposure duration, and estimated melting-related product loss in the investigated crystal ice distribution system? (2) How does the lateness-penalty parameter (λ) influence the trade-off between routing efficiency and customer-service punctuality, and what is the minimum tested λ value that produces a zero-lateness solution while maintaining vehicle-capacity and depot-return feasibility?

Methods

Research Type

This study used a quantitative computational case-study approach within the field of operations research. The study was conducted to evaluate crystal ice distribution routes by applying the Vehicle Routing Problem with Time Windows (VRPTW) under hard and penalty-based soft time-window conditions (Figliozzi, 2010; X. Liu et al., 2023; Toth & Vigo, 2014). The route construction process was solved using the Cheapest Insertion Heuristic (CIH).

The operational data were collected during October 2025 through direct observation of the company's distribution activities over ten delivery days. The distribution system consisted of two non-refrigerated insulated box trucks, each with a maximum physical capacity of 3,000 kg. Both vehicles departed from the depot at 07:00 and were required to return before the end of the company's regular distribution period at 17:00. No separate driver working hour constraint was imposed beyond this operating horizon. Therefore, the 07:00 – 17:00 depot operating period effectively limited each vehicle route to a maximum duration of 10 hours. The distribution network consisted of one depot and 60 scheduled customer delivery nodes with different demand quantities and service time windows. Customer service duration was determined dynamically according to customer demand using $s_i = \lceil 5 + \frac{q_i}{1} \rceil$ minutes, where s_i represents the service duration at customer i and q_i represents customer demand in kilograms. The calculated service duration was rounded upward to the nearest whole minute and incorporated into the route-time and time-window feasibility calculations. Data preprocessing consisted of anonymizing customer identifiers and coordinates and standardizing time-window values into a consistent computational time format.

The company's existing customer allocation and delivery sequences were used as the operational baseline. During optimization, however, customer allocation and route

sequences were allowed to change between the two vehicles, provided that each customer was served exactly once and the

vehicle capacity, customer time window, and depot return

Table 2. Customer Demand and Service Time-Window Data

Node	Location_Type	Demand(kg)	Earliest Service	Latest Service
D0	Depot	0	7:00	17:00
C01	Customer	80	8:00	16:00
C02	Customer	70	8:00	13:00
C03	Customer	40	7:00	10:00
C04	Customer	35	7:00	9:30
C05	Customer	95	7:00	10:00
C06	Customer	35	7:00	9:30
C07	Customer	45	7:30	9:30
C08	Customer	35	8:00	12:00
C09	Customer	75	8:00	16:00
C10	Customer	40	14:30	16:00
C11	Customer	20	6:00	9:00
C12	Customer	45	7:30	8:30
C13	Customer	35	7:00	8:00
C14	Customer	35	7:00	10:00
C15	Customer	50	7:00	10:00
C16	Customer	80	8:00	16:00
C17	Customer	70	9:00	12:00
C18	Customer	45	8:00	16:00
C19	Customer	55	6:30	9:00
C20	Customer	30	9:00	11:30
C21	Customer	85	8:00	14:00
C22	Customer	75	8:00	16:30
C23	Customer	85	8:30	13:00
C24	Customer	80	9:30	12:00
C25	Customer	90	8:00	16:00
C26	Customer	45	8:00	10:00
C27	Customer	75	8:00	14:30
C28	Customer	75	7:00	15:00
C29	Customer	80	12:00	17:00
C30	Customer	30	7:00	11:30
C31	Customer	40	9:00	12:00
C32	Customer	80	9:00	15:00
C33	Customer	20	13:30	16:00
C34	Customer	75	7:30	16:30
C35	Customer	85	8:00	16:30
C36	Customer	35	7:00	10:00
C37	Customer	20	8:00	10:30
C38	Customer	25	8:30	12:00
C39	Customer	60	9:00	15:00
C40	Customer	30	14:00	16:30
C41	Customer	75	8:30	15:30
C42	Customer	60	7:30	16:00
C43	Customer	35	7:00	9:00
C44	Customer	50	9:00	17:00
C45	Customer	25	8:00	16:30
C46	Customer	35	7:30	11:00
C47	Customer	35	7:00	12:30
C48	Customer	20	9:00	12:00
C49	Customer	15	8:00	9:30
C50	Customer	30	8:00	11:00
C51	Customer	70	7:00	15:00
C52	Customer	35	7:00	12:00
C53	Customer	20	8:00	11:00
C54	Customer	60	7:00	16:00
C55	Customer	20	7:00	12:30
C56	Customer	65	7:00	16:30
C57	Customer	70	7:00	17:00
C58	Customer	65	10:00	17:00
C59	Customer	20	8:30	11:00
C60	Customer	40	7:00	15:30

constraints were satisfied. The main model inputs included customer demand, service time windows, service duration, vehicle capacity, depot location, and road-network distance and travel-time matrices.

Population and Sample/Informants

The primary unit of analysis in this study was the distribution-routing system rather than a human population, and no statistical sampling of individuals was conducted. The unit of analysis consisted of one depot, customer delivery nodes, vehicle capacity, customer demand, service time windows, the initial delivery routes, and the distance and travel-time matrices between distribution points. Company personnel were consulted only to provide supplementary clarification regarding operational loading and distribution practices and were not treated as statistical sampling units. To maintain confidentiality, customer names and exact coordinates were anonymized and represented using node codes. The depot was coded as D0, while customers were coded as C01, C02, C03, and so forth.

Research Location

The study was conducted in the crystal ice distribution network of a small-scale distributor located in the Jimbaran–Uluwatu area, Bali, Indonesia. This area was selected because the company distributes crystal ice to multiple customers using non-refrigerated vehicles. Under this operating condition, product preservation depends mainly on vehicle insulation, route sequence, delivery duration, and the ability to meet customer service time windows.

Research Object and Routing Data

The routing data used in this study consisted of one depot and customer delivery nodes. Each customer node was represented using an anonymized code to protect customer identity and commercial confidentiality. The main input variables included customer demand and service time windows, which were used to evaluate vehicle-capacity feasibility and delivery-time reliability. The anonymized customer demand and service time-window data are presented in Table 2.

Instrumentation or Tools

Road-network distance and travel-time matrices were generated from OpenStreetMap-based OpenRouteService (ORS) using the driving-car profile. The 61×61 matrices covered the depot (D0) and 60 customer nodes (C01–C60) and were generated in November 2025, approximately one month after the October 2025 operational observation period; the exact API retrieval day was not retained in the analytical archive. Coordinate inputs were stored to approximately six to seven decimal places in the working data. The matrices represented static road-network estimates and did not incorporate real-time traffic information or time-dependent traffic conditions. For the routing calculations, the retained distance and travel-time matrices were treated as symmetric and time-invariant. Thus, the reported travel times should be interpreted as static ORS road-network estimates rather than observed traffic-dependent travel times.

Data Collection Procedures

Data were collected from the company's distribution operation. The collected data consisted of customer locations, customer demand, customer service time windows, vehicle capacity, initial delivery routes, and product-loss observations. The initial delivery route was obtained from the company's existing distribution practice and used as a baseline for comparison. Customer locations were converted into anonymized node data and used to generate real road-network distance and travel time matrices. All time-window

values were standardized using the 24-hour HH format and converted into minutes from the start of the planning horizon for computational processing. Supplementary operational information was also obtained from company personnel during the field observation period to clarify loading and distribution practices, including the handling of additional crystal ice beyond scheduled customer orders.

Data Model Development

The data processing stage started by coding all distribution points into anonymized nodes. The depot was defined as the start and end point of each vehicle route. Customer demand and vehicle capacity were used to test route feasibility. Customer time windows were expressed as the earliest service time and latest service time for each customer. The distance matrix represented the travel distance between every pair of nodes, while the travel-time matrix represented the estimated travel duration between every pair of nodes.

Mathematical Formulation of VRPTW Models

The routing problem was formulated as a Vehicle Routing Problem with Time Windows (VRPTW). The model consists of a set of customers, vehicles, and depot nodes. The objective was to minimize total travel distance while considering vehicle capacity, customer service requirements, and time-window constraints.

$N = \{0, 1, 2, \dots, n\}$

where:

- 0 represents the depot
- $1, \dots, n$ represent customer nodes

$K = \{1, 2, \dots, m\}$

where:

- K represents available vehicles.

Symbol	Description
c_{ij}	travel distance from node i to j
t_{ij}	travel time from node i to j
q_i	customer demand
Q	vehicle capacity
a_i, b_i	earliest and latest service time
s_i	service duration
λ	lateness penalty parameter

The model uses routing, service time, lateness, and auxiliary ordering variables. The variable T_i represents the service starting time at customer node i, while the binary routing variable x_{ijk} indicates whether vehicle k travels directly from node i to node j.

The decision variable x_{ijk} is defined as:

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ travels from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases}$$

$$x_{ijk} \in \{0, 1\}, \quad \forall i, j \in N, i \neq j, k \in K$$

For schedule reconstruction, A_i denotes the arrival time at customer i, W_i denotes waiting time, T_i denotes service-start time, and s_i denotes service duration. If a vehicle arrives before the earliest allowable service time a_i , waiting time is calculated as

$$W_i = \max(0, a_i - A_i)$$

and services start at:

$$T_i = A_i + W_i = \max(A_i, a_i)$$

Customer lateness is defined as

$$L_i = \max(0, T_i - b_i)$$

where b_i is the latest allowable service start time. Service completion occurs at $T_i + s_i$ and this completion time is used in the time propagation to the next node. For vehicle k, route duration is measured from depot departure to depot return and

includes travel time, waiting time, and customer-service duration. The auxiliary ordering variable u_{ik} represents the visit order of customer i in the route of vehicle k and is used for subtour elimination.

Hard Time Window Model

$$\text{Min } Z = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ijk} \\ a_i \leq T_i \leq b_i$$

Soft Time Window Model

$$\text{Min}_{x,T,L,u} Z_S = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ijk} + \lambda \sum_{i \in N \setminus \{0\}} L_i$$

In the soft objective function, the first term represents total travel distance, while the second term represents the aggregate customer lateness penalty. The lateness penalty is applied to each customer through L_i , and the depot is excluded from the lateness summation.

When a vehicle arrives before the earliest allowable service time, the vehicle is required to wait until the beginning of the customer's time window. The waiting time does not contribute to lateness calculation because lateness is only measured when the service start time exceeds the latest allowable service time. The service duration at each customer node is incorporated into the route time calculation through the time propagation constraint. After completing all customer deliveries, each vehicle returns to the depot, and the return travel time is included in the total route distance and travel-time calculation.

The lateness penalty parameter (λ) was introduced to balance travel-distance efficiency and delivery-time reliability. In this study, λ is expressed in km/min, while customer lateness is expressed in minutes. Therefore, the product of λ and lateness produces a distance-equivalent penalty in kilometers. Consequently, the travel-distance and lateness-penalty components of the soft objective function are evaluated on the same generalized-distance scale. A lower λ gives greater priority to distance reduction, whereas a higher λ gives greater priority to reducing customer lateness.

$$\sum_{i \in N \setminus \{0\}} q_i \left(\sum_{j \in N} x_{ijk} \right) \leq Q, \forall k \in K$$

This constraint ensures that the total customer demand assigned to each vehicle does not exceed the available vehicle capacity.

$$\sum_{k \in K} \sum_{j \in N} x_{ijk} = 1, \quad \forall i \in N \setminus \{0\}$$

This constraint ensures that each customer node is visited exactly once by one vehicle.

$$\sum_{j \in N} x_{ijk} = \sum_{j \in N} x_{jik}, \quad \forall i \in N \setminus \{0\}, k \in K$$

The flow conservation constraint ensures that each vehicle entering a customer node must also leave the node.

Because the study used a fixed fleet of two vehicles, both vehicles were included in the route-construction process. Each vehicle was therefore required to depart from the depot once and return to the depot once. The corresponding depot and vehicle-use constraints are expressed as:

$$\sum_{j \in N} x_{0jk} = 1, \forall k \in K \\ \sum_{i \in N} x_{i0k} = 1, \forall k \in K$$

The first constraint ensures that each vehicle departs from the depot exactly once, while the second constraint

ensures that each vehicle returns to the depot exactly once after completing its assigned deliveries.

$$T_j \geq T_i + s_i + t_{ij} - M(1 - x_{ijk})$$

This constraint maintains the chronological consistency of the route by considering travel time and service duration between consecutive customer visits, where M represents a sufficiently large positive constant used to relax the time constraint when the route between nodes i and j is not selected.

To prevent the formation of disconnected customer cycles that do not include the depot, the auxiliary ordering variable u_{ik} was used in the Miller-Tucker-Zemlin (MTZ) subtour-elimination constraint, expressed as:

$$u_{ik} - u_{jk} + nx_{ijk} \leq n - 1, \quad \forall i, j \in N \setminus \{0\}, i \neq j, k \in K$$

$$1 \leq u_{ik} \leq n, \quad \forall i \in N \setminus \{0\}, k \in K$$

This constraint prevents the formation of a closed customer cycle that is disconnected from the depot and ensures that the customer sequence of each vehicle remains part of a continuous depot-to-depot route.

In addition to customer time-window feasibility, each vehicle was required to return to the depot no later than the end of the company's regular distribution period at 17:00. Let b_0 denote the latest allowable depot-return time, where $b_0 = 17:00$. For the final customer i served by vehicle k , the depot-return constraint is expressed as:

$$T_i + s_i + t_{i0} \leq b_0 + M(1 - x_{i0k}), \quad \forall i \in N \setminus \{0\}, k \in K$$

This constraint ensures that the service at the final customer, together with the travel time back to the depot, does not cause the vehicle to return after 17:00. The depot-return requirement was treated as a hard feasibility constraint in both the hard and soft time-window models.

Two routing structures were evaluated. The first was a hard time-window VRPTW model, in which any insertion causing customer lateness was considered infeasible and rejected. The second was a penalty-based soft time-window VRPTW model, in which lateness remained feasible but was incorporated into the insertion cost through the penalty parameter λ . Both models used the same customer-demand, vehicle-capacity, service-time, distance, and travel-time data.

Cheapest Insertion Heuristic Procedure

The Cheapest Insertion Heuristic was used to construct vehicle routes iteratively. Each vehicle route was initialized as depot-depot, and all customers were initially marked as unserved. During each iteration, the algorithm evaluated every possible insertion of an unserved customer into the current vehicle routes. The customer was inserted into the position that produced the lowest additional cost while still considering vehicle capacity and time-window conditions. This insertion principle follows classical route-construction heuristics that add unserved nodes according to incremental travel cost (Rosenkrantz et al., 1977).

The basic insertion cost of inserting customer u between node i and node j was calculated as:

$$\Delta c = c_{iu} + c_{uj} - c_{ij}$$

where Δc represents the additional travel distance caused by inserting customer u between two consecutive nodes.

For the penalty-based soft time-window model, the insertion cost was extended as follows:

$$\Delta C = \Delta c + \lambda \Delta L$$

where ΔC represents the total insertion cost, Δc represents the additional travel distance, λ represents the lateness penalty coefficient expressed in km/min, and ΔL represents the change in total lateness after insertion.

In the hard time-window scenario, insertions that created lateness were rejected. In the soft time-window scenario, lateness was allowed but included in the insertion cost through the penalty term. This mechanism enabled the algorithm to select routes that were not only shorter but also more reliable in terms of service time.

Cheapest Insertion Heuristic for VRPTW with Lateness Penalty

The solution construction phase employs a cheapest insertion heuristic adapted for the VRPTW with lateness penalty (Algorithm 1).

Penalty Parameter Selection

The lateness penalty parameter, λ , was used to control the trade-off between travel-distance efficiency and delivery punctuality. A smaller λ value makes the algorithm prioritize shorter distance, while a larger λ value makes the algorithm prioritize lower lateness. The penalty parameter was selected through sensitivity testing by running the same CIH procedure under several λ values.

The tested λ values were 0.1, 0.5, 1, 2, and 5 km/min. The penalty coefficient was selected using a minimum sufficient penalty rule, defined as the smallest tested λ that eliminated all customer lateness while maintaining vehicle-capacity and depot-return feasibility. The same routing procedure and operational constraints were applied to all λ values to ensure a consistent sensitivity comparison.

Product Loss Estimation

Product loss was estimated using a linear proportional model calibrated from field observations under the company's regular distribution operations. The calibration data were obtained over ten delivery days. During this observation period, the average initial physical loads were 1,910.5 kg for Truck 1 and 2,055 kg for Truck 2, while the average baseline distribution exposure duration was approximately 600 minutes per truck. Before departure, the physical load carried by each truck was weighed and recorded, including scheduled customer orders and additional crystal ice carried for operational purposes. After completing the delivery route, the remaining physical load was weighed again upon return to the depot. Scheduled deliveries and incidental retail sales made during the route were recorded as part of the company's regular operational procedures. Based on the company's reconciled operational records, the observed average melting-related losses were 269 kg for Truck 1 and 311 kg for Truck 2. The corresponding company-reported

Algorithm 1. Cheapest Insertion Heuristic for VRPTW with Lateness Penalty

Input: Customer set C , depot O , vehicle capacity Q , customer demand q_i , distance matrix d_{ij} , travel-time matrix t_{ij} , service time s_i , time windows $[a_i, b_i]$, and penalty parameter λ .

Output: Final vehicle routes, total distance, total travel time, total lateness, and feasibility status.

```

1: Initialize each vehicle route as depot  $\rightarrow$  depot.
2: Set all customers as unserved.
3: while unserved customers remain do
    4: bestInsertion  $\leftarrow \emptyset$ 
    5: bestInsertionCost  $\leftarrow \infty$ 
    6: for each vehicle route  $r$  do
        7: for each unserved customer  $i$  do
            8: if adding customer  $i$  exceeds vehicle capacity  $Q$  then
                9: continue
            10: end if
            11: for each possible insertion position  $p$  in route  $r$  do
                12: Temporarily insert customer  $i$  at position  $p$ .
                13: Recalculate route distance, schedule, and lateness.
                14: Compute the incremental insertion cost:  $\Delta C = \Delta d + \lambda \Delta L$ .
                15: if the hard time-window model is applied and the insertion violates a time-window constraint then
                    16: Reject the insertion.
                    17: continue
                18: end if
                19: if the soft time-window model is applied then
                    20: Allow the insertion and incorporate lateness through the penalty term  $\lambda \Delta L$ .
                21: end if
                22: if  $\Delta C < \text{bestInsertionCost}$  then
                    23: bestInsertionCost  $\leftarrow \Delta C$ 
                    24: bestInsertion  $\leftarrow (r, i, p)$ 
                25: end if
            26: end for
        27: end for
    28: end for
    29: if bestInsertion =  $\emptyset$  then
        30: Stop and report infeasibility.
    31: end if
    32: Apply bestInsertion to the selected vehicle route.
    33: Update the vehicle load and route schedule.
    34: Remove the inserted customer from the unserved customer set.
35: end while
36: Return the final routes and route-performance indicators.

```

Table 3. Baseline Operational Characteristics and Observed Product Loss

Vehicle	Average Initial Load (kg)	Baseline Duration (min)	Baseline Distance (km)	Observed Loss (kg)	Reported Loss (%)
Truck 1	1910.5	600	52.50	269	14.09
Truck 2	2055	600	47.34	311	15.13

loss proportions were approximately 14.09% and 15.13%, respectively, relative to the initial physical loads. These percentages are presented as descriptive baseline information. The product-loss estimation model itself was calibrated directly using the observed average loss masses of 269 kg and 311 kg and the corresponding baseline route exposure durations, rather than using the percentage values as its primary input (Table 3).

The observed initial physical load should not be interpreted as equivalent to the scheduled customer demand used in the VRPTW model. The routing instance represents 3,050 kg of predetermined demand assigned to the 60 scheduled customer nodes. Based on information provided by company personnel during the field observation, the trucks may depart with additional crystal ice beyond the scheduled customer orders. This additional quantity is carried as a buffer against melting-related shrinkage during distribution and as inventory for incidental retail sales that may occur along the delivery route. Because these additional quantities are not assigned to the scheduled customer nodes, they are not included in the modeled VRPTW demand. Consequently, the observed initial physical load represents the total quantity physically carried by the truck at departure, whereas the modeled customer demand represents only the predetermined quantities assigned to the 60 scheduled customers.

The truck-specific product-loss rate was calibrated directly from the observed average loss mass and the baseline route exposure duration as follows:

$$r_k = \frac{L_{obs,k}}{T_{obs,k}}$$

where r_k represents the estimated product-loss rate for vehicle k in kg/minute, $L_{obs,k}$ represents the observed average product-loss mass for vehicle k , and $T_{obs,k}$ represents the baseline route exposure duration.

$$\widehat{L}_{k,s} = r_k \times T_{k,s}$$

where $\widehat{L}_{k,s}$ represents the model-estimated product loss for vehicle k under routing scenario s , and $T_{k,s}$ represents the full route exposure duration of vehicle k under scenario s .

Accordingly, the product-loss values obtained from the ten-day field observations were treated as observed baseline losses, whereas the product-loss values reported for the optimized routing scenarios were model-estimated values. The estimation assumes a linear proportional relationship between distribution duration and product loss within the operational range considered. Therefore, reductions in distribution duration produce proportional reductions in estimated product loss. Other factors that may influence ice melting, including ambient temperature, loading and unloading conditions, handling practices, and variations in vehicle insulation performance, were not explicitly incorporated into the estimation model and are acknowledged as limitations of the study.

Data Analysis

The analysis compared the company's baseline distribution condition, the regenerated hard time-window solution, and the penalty-based soft time-window solutions generated under $\lambda = 0.1, 0.5, 1, 2$, and 5 km/min. The hard time-window solution was used as the zero-lateness reference, while the soft time-window scenarios were evaluated to examine the trade-off between routing efficiency and customer-service punctuality under different lateness-penalty values.

The main performance indicators were total travel distance, pure travel time, route exposure duration, total lateness, number of late customers, vehicle-capacity feasibility, depot-return feasibility, and estimated product loss. Travel distance and pure travel time were used to evaluate routing efficiency, whereas total lateness and the

number of late customers were used to evaluate service-time reliability. Route exposure duration included travel time, waiting time, and customer-service time and was used as the basis for estimating melting-related product loss.

The soft time-window penalty was selected using the minimum sufficient penalty rule, defined as the smallest tested λ value that eliminated all customer lateness while maintaining vehicle-capacity and depot-return feasibility. Accordingly, route evaluation was not based solely on the shortest distance. The final assessment considered the trade-off between distance efficiency, service punctuality, operational feasibility, and product exposure duration under the investigated distribution conditions.

Verification and Validation

Verification was conducted to ensure that the algorithm followed the intended routing logic. The verification process checked whether every route started and ended at the depot, whether each customer was served exactly once, whether vehicle capacity was not exceeded, whether arrival time and service start time were calculated sequentially, and whether lateness was calculated correctly after each insertion. For the hard time-window scenario, lateness-causing insertions were rejected, whereas for the soft time-window scenario, lateness was incorporated into the insertion-cost calculation through the penalty term.

As an independent feasibility check, the final optimized routes were reconstructed separately from the route-construction procedure using a post-processing verification script that did not perform customer insertion or local-search optimization. Each route was evaluated sequentially from the depot by recalculating travel time, arrival time, waiting time, service start time, service duration, cumulative vehicle load, lateness, and return to the depot. The independent reconstruction reproduced the reported route totals with zero difference within floating-point precision and confirmed compliance with the vehicle-capacity and depot-return constraints. Detailed route-level feasibility checks are presented together with the routing results.

The Cheapest Insertion Heuristic implementation used in this study was deterministic; no random initialization or stochastic operator was applied. Therefore, repeated runs using identical input data and parameter settings produce the same routing solution, and stochastic repeated run analysis was not applicable. Direct validation of the OpenRouteService travel time estimates against customer level GPS or observed point to point travel time records could not be performed because such field records were not available. Accordingly, the travel time values used in this study should be interpreted as static road network estimates rather than field validated travel times. Similarly, the ten day product loss observations were used to calibrate the exposure based loss model and were not divided into separate calibration and validation samples. Product loss values for the optimized scenarios should therefore be interpreted as model estimated values rather than independently field validated losses.

To provide an additional exact-solution benchmark, a reduced six-customer instance was constructed using C52, C43, C19, C13, C11, and C12, corresponding to the first six customer nodes in the final Truck 2 hard time-window route. Because the reduced instance contained only six customers, all $6! = 720$ possible customer sequences could be evaluated by exhaustive enumeration using the same ORS distance and travel time matrices, customer service durations, time windows, vehicle capacity, depot departure time, and depot-return requirement as the main model. Of the 720 possible sequences, 293 satisfied all hard time window and operational feasibility requirements. The exact minimum feasible distance was 36.46 km. Applying the same deterministic Cheapest Insertion Heuristic to the reduced instance also produced a

feasible route with a total distance of 36.46 km, identical to the exact minimum distance, with zero lateness, a total scheduled demand of 225 kg, and a depot return at approximately 08:46. Thus, the heuristic achieved a 0.00% optimality gap relative to the exhaustive exact solution for this reduced benchmark. This benchmark is intended only as a verification of the heuristic implementation on a tractable small instance and does not establish global optimality for the full 60-customer routing problem.

In addition to the independent feasibility check, the optimized routes were compared with the company's initial routes to evaluate operational performance. The comparison focused on total travel distance, total travel time, lateness, and estimated product loss. Therefore, the baseline comparison was treated as a performance evaluation rather than as the sole method of model validation.

Formal practitioner validation of the optimized route sequences was not conducted. Company personnel were consulted to clarify loading and distribution practices but were not asked to formally approve the optimized routes. Accordingly, operational feasibility in this study refers to computational compliance with the modeled vehicle-capacity, customer time-window, and depot-return constraints using the adopted road-network distance and travel-time inputs. Customer specific informal preferences, driver familiarity, and operational road-access conditions not represented in the routing data were not independently confirmed. Therefore, the proposed routes should be field-tested before full operational implementation.

The optimization procedure was implemented in Python using Jupyter Notebook. The same customer-demand data, vehicle-capacity parameters, time-window data, service-duration data, and road-network distance and travel-time matrices were used as model inputs. Given the same input data, parameter settings, and Cheapest Insertion Heuristic procedure, the computational process can be rerun to reproduce the reported routing results. The verification code and supporting routing data are provided as supplementary materials to facilitate computational reproducibility.

Ethical Considerations

This study primarily used company operational data for computational routing analysis. Company personnel were consulted only to provide supplementary clarification regarding loading and distribution practices; no personal, clinical, or sensitive information from company personnel was collected or reported. Customer names, exact addresses, and coordinates were anonymized, and the published data were presented using node codes to protect customer identity and commercial confidentiality.

Result and Discussion

Routing Data Characteristics

The routing data used in this study consisted of one depot and sixty customer delivery nodes. Each customer node had a specific demand value and service time window. The depot was coded as D0, while customer locations were anonymized using node codes from C01 to C60. Customer demand ranged from 15 kg to 95 kg, while the service time windows varied according to customer availability. These data were used as the main input for evaluating route feasibility, vehicle capacity utilization, and delivery-time reliability. The distribution routes were constructed using real road-network distance and travel-time matrices. The use of real road-network data allowed the model to represent actual vehicle movement more realistically than straight-line distance estimation. The matrices were used to calculate route distance, travel time, arrival time, service start time, and lateness for each routing scenario.

Sensitivity Analysis of the Lateness Penalty Parameter

As shown in Table 4, the sensitivity analysis shows that $\lambda = 0.1$ km/min produced the shortest fleet distance of 85.29 km but resulted in 8.95 minutes of total lateness across two customers. Increasing λ to 0.5 km/min eliminated all customer lateness, with the fleet distance increasing to 86.87 km. Further increases to $\lambda = 1, 2,$ and 5 km/min produced the same distance and zero-lateness solution. Therefore, $\lambda = 0.5$ km/min was selected as the minimum sufficient penalty. The unchanged solution for $\lambda \geq 0.5$ indicates that the recommendation is robust within the tested penalty range.

Hard Time-Window Routing Results

The hard time-window scenario required all scheduled customers to be served within their specified service time windows. Any route insertion that resulted in positive customer lateness was therefore rejected. Using the regenerated routing data, the hard time-window model successfully served all 60 scheduled customers with the two available vehicles while satisfying the vehicle-capacity, customer time-window, and depot-return constraints.

The resulting fleet travel distance was 86.87 km with zero customer lateness. Truck 1 served 1,185 kg of scheduled demand over a distance of 30.99 km, while Truck 2 served 1,865 kg over a distance of 55.88 km. Both vehicle loads remained below the maximum physical capacity of 3,000 kg per truck.

The total pure travel time for the fleet was 123.09 minutes, consisting of 44.90 minutes for Truck 1 and 78.19 minutes for Truck 2. In addition, the routes included 620.00 minutes of customer service time and approximately 300.50 minutes of waiting time. The resulting total route exposure duration was approximately 1,043.59 minutes for the two vehicles combined. Both trucks departed from the depot at 07:00 and returned before the depot closing time of 17:00.

These results confirm that the regenerated hard time-window solution was feasible under the stated operational constraints and provided a zero-lateness reference solution for comparison with the penalty-based soft time-window scenarios.

Soft Time-Window Routing Results

The soft time-window scenario allowed customer lateness to remain feasible but incorporated the resulting violation into the route insertion cost through the lateness penalty parameter, λ . This formulation enabled the algorithm to evaluate the trade-off between shorter travel distance and customer-service punctuality instead of automatically rejecting every insertion that produced lateness.

The routing results varied according to the magnitude of the penalty parameter. At $\lambda = 0.1$ km/min, the soft time-window model produced the shortest fleet travel distance of 85.29 km. However, this reduction in distance was accompanied by 8.95 minutes of total lateness across two customers. The individual lateness values were approximately 1.57 minutes and 7.38 minutes. Therefore, the lateness observed at $\lambda = 0.1$ was treated as an actual time-window violation rather than a numerical rounding effect.

When the penalty was increased to $\lambda = 0.5$ km/min, all customer lateness was eliminated. The resulting fleet distance was 86.87 km, identical to the distance obtained under the hard time-window model. The customer allocation and route

Table 4. Sensitivity Analysis of the Lateness Penalty Parameter

λ	Fleet Distance (km)	Total Lateness (min)	Late Customers
0.1	85.29	8.95	2
0.5	86.87	0.00	0
1	86.87	0.00	0
2	86.87	0.00	0
5	86.87	0.00	0

sequences at $\lambda = 0.5$ were also identical to those of the regenerated hard time-window solution. Increasing the penalty further to $\lambda = 1, 2$, and 5 km/min produced the same 86.87-km zero-lateness solution and therefore provided no additional improvement.

Accordingly, $\lambda = 0.5 \text{ km/min}$ was selected as the minimum sufficient penalty, defined as the smallest tested penalty value that eliminated customer lateness while maintaining the vehicle-capacity and depot-return constraints. Compared with $\lambda = 0.1$, the selected $\lambda = 0.5$ solution required an additional 1.58 km of fleet travel distance but eliminated 8.95 minutes of total lateness. This result demonstrates the operational trade-off between marginal distance savings and customer-service punctuality in the investigated distribution system.

Lateness was calculated using the numerical precision of the travel-time and service-time data and was rounded to two decimal places only for presentation. Any positive lateness value was treated as a time-window violation, and no artificial tolerance threshold was used to convert small positive values to zero.

Route-Level Feasibility Verification

To provide transparent route-level verification, the complete customer sequences of the final selected solution were examined for both vehicles. Under the regenerated results, the hard time-window solution and the selected soft time-window solution at $\lambda = 0.5 \text{ km/min}$ produced the same zero-lateness route.

For Truck 1, the final route sequence was:

D0 → C60 → C59 → C55 → C49 → C35 → C36 → C37 → C38 → C46 → C26 → C47 → C48 → C27 → C53 → C50 → C51 → C30 → C31 → C34 → C32 → C33 → C29 → C28 → C54 → C56 → C57 → D0

For Truck 2, the final route sequence was:

D0 → C52 → C43 → C19 → C13 → C11 → C12 → C15 → C06 → C04 → C07 → C05 → C03 → C14 → C18 → C20 → C23 → C24 → C22 → C21 → C17 → C16 → C01 → C02 → C08 → C09 → C10 → C40 → C39 → C41 → C42 → C25 → C44 → C45 → C58 → D0

As shown in Table 5, both routes departed from the depot at 07:00 and satisfied the $3,000\text{-kg}$ vehicle-capacity limit, customer time-window requirements, and the depot-return constraint. The complete customer-by-customer schedule, including demand, cumulative load, leg distance, travel time, arrival time, earliest and latest service times, waiting time, service start, service duration, service end, lateness, capacity status, and time-window feasibility status, is provided in the Supplementary Material.

As an additional load-consistency check, the scheduled demand served by each vehicle was compared with the model-estimated product loss. Truck 1 served $1,185 \text{ kg}$ of scheduled customer demand and had an estimated product loss of 211.22 kg , corresponding to a combined demand plus estimated-loss quantity of $1,396.22 \text{ kg}$. Truck 2 served $1,865 \text{ kg}$ of scheduled demand and had an estimated product loss of 296.73 kg , corresponding to a combined demand-plus-estimated-loss quantity of $2,161.73 \text{ kg}$. Both quantities remain below the $3,000\text{-kg}$ physical capacity of each vehicle. At the fleet level, the $3,050 \text{ kg}$ of scheduled customer demand plus the estimated loss of 507.95 kg corresponds to a combined demand plus estimated loss quantity of $3,557.95 \text{ kg}$, which remains below the combined fleet capacity of $6,000 \text{ kg}$. The estimated fleet loss of 507.95 kg represents a reduction of 72.05 kg , or 12.42% , relative to the observed baseline loss of 580 kg .

Comparative Performance of Routing Scenarios

The comparative analysis considered the company's existing distribution condition, the regenerated hard time-window solution, and the soft time-window solutions at different lateness-penalty levels. Because the selected soft time-window solution at $\lambda = 0.5 \text{ km/min}$ converged to the same customer allocation and route sequence as the hard time-window solution, the comparison was conducted primarily at the fleet level rather than by identifying a different preferred routing rule for each vehicle.

Table 6 shows that, for Truck 1, the low-penalty soft time-window scenario at $\lambda = 0.1$ did not improve route performance relative to the hard time-window solution. The soft route produced a slightly longer travel distance of 31.22 km compared with 30.99 km under the hard model and generated 1.57 minutes of lateness. Pure travel time also increased slightly from 44.90 to 45.16 minutes. However, both scenarios resulted in the same route exposure duration of 471.12 .

Table 5. Route-Level Feasibility Summary of the Final Selected Solution

Indicator	Truck 1	Truck 2
Demand served (kg)	1,185	1,865
Distance (km)	30.99	55.88
Pure travel time (min)	44.90	78.19
Service time (min)	254.00	366.00
Waiting time (min)	172.23	128.27
Route exposure duration (min)	471.12	572.46
Total Lateness (min)	0.00	0.00
Capacity constraint (3,000 kg)	Satisfied	Satisfied
Depot return (17:00)	Satisfied	Satisfied

Table 6. Performance Comparison of Soft-Constraint VRPTW ($\lambda = 0.1$) and Hard-Constraint VRPTW for Truck 1

Evaluation Criteria	Unit	Soft-Constraints VRPTW ($\lambda = 0.1$)	Hard-Constraints VRPTW	Better Model
Total Lateness	minutes	1.57	0.00	Hard-Constraints
Pure Travel Time	minutes	45.16	44.90	Hard-Constraints
Total Travel Distance	km	31.22	30.99	Hard-Constraints
Route Exposure Duration	minutes	471.12	471.12	Equal
Estimated Product Loss	kg	211.22	211.22	Equal

Table 7. Performance Comparison of Soft-Constraint and Hard-Constraint VRPTW for Truck 2

Evaluation Criteria	Unit	Soft-Constraints VRPTW ($\lambda = 0.1$)	Hard-Constraints VRPTW	Better Model
Total Lateness	minutes	7.38	0.00	Hard-Constraints
Pure Travel Time	minutes	75.98	78.19	Soft-Constraints
Total Travel Distance	km	54.07	55.88	Soft-Constraints
Route Exposure Duration	minutes	572.46	572.46	Equal
Estimated Product Loss	kg	296.73	296.73	Equal

minutes and, consequently, the same estimated product loss of 211.22 kg. Therefore, the hard time-window solution provided better service punctuality and slightly better routing efficiency for Truck 1 at $\lambda = 0.1$, while no difference was observed in the exposure-based product-loss estimate.

The comparison for Truck 2 is presented in Table 7.

Table 7 shows a different trade-off for Truck 2. At $\lambda = 0.1$, the soft time-window scenario reduced travel distance from 55.88 to 54.07 km and pure travel time from 78.19 to 75.98 minutes. However, these efficiency gains were accompanied by 7.38 minutes of customer lateness. Despite the differences in travel time and route sequence, both scenarios resulted in the same route exposure duration of 572.46 minutes and, consequently, the same estimated product loss of 296.73 kg. Thus, the soft model provided modest distance and pure-travel-time savings for Truck 2, but these benefits were achieved at the cost of reduced service punctuality.

As shown in Table 8, At the fleet level, the low-penalty soft time-window scenario at $\lambda = 0.1$ produced a total travel distance of 85.29 km, compared with 86.87 km under the hard time-window model. This represents a distance reduction of 1.58 km, or approximately 1.82%. Pure travel time also decreased from 123.09 minutes under the hard model to 121.14 minutes under the soft model. However, these efficiency gains were accompanied by 8.95 minutes of total lateness across two customers, whereas the hard time-window model maintained zero lateness.

Despite the differences in travel distance, pure travel time, and service punctuality, both scenarios resulted in approximately the same total route exposure duration of 1,043.59 minutes and consequently the same model-estimated fleet product loss of 507.95 kg. This result indicates that the modest reduction in pure travel time under $\lambda = 0.1$ was offset by differences in waiting and service timing, so the total period during which the crystal ice remained inside the non-refrigerated vehicles did not decrease.

When the penalty parameter was increased to $\lambda = 0.5$ km/min, all customer lateness was eliminated and the soft time-window solution converged to the same 86.87-km route as the hard time-window solution. Therefore, $\lambda = 0.5$ was retained as the minimum sufficient penalty for the final soft time-window setting.

Figure 1 and Figure 2 present the optimized route structures generated under the hard time-window model and the low-penalty soft time-window model at $\lambda = 0.1$ km/min. The hard time-window solution enforced complete compliance with customer service time windows and produced a fleet distance of 86.87 km with zero lateness. In contrast, the soft time-window solution at $\lambda = 0.1$ allowed lateness to be penalized rather than automatically rejected, resulting in a different customer allocation and route sequence. The soft scenario reduced the fleet distance to 85.29 km and pure travel time to 121.14 minutes, compared

with 86.87 km and 123.09 minutes under the hard model, but generated 8.95 minutes of total lateness across two customers. Therefore, the route visualizations should be interpreted together with the performance indicators, because differences in route structure reflect a trade-off between travel efficiency and service punctuality rather than route shape alone.

At the vehicle level, the effects of the low-penalty soft time-window setting differed between the two trucks. For Truck 1, the soft scenario at $\lambda = 0.1$ produced a slightly longer travel distance of 31.22 km compared with 30.99 km under the hard model and generated 1.57 minutes of lateness. Pure travel time also increased slightly from 44.90 to 45.16 minutes. For Truck 2, the soft scenario reduced travel distance from 55.88 to 54.07 km and pure travel time from 78.19 to 75.98 minutes, but generated 7.38 minutes of lateness. Despite these differences in routing efficiency and service punctuality, the route exposure duration and estimated product loss remained unchanged for both vehicles because changes in travel time were offset by differences in waiting and service timing.

At the fleet level, the soft time-window scenario at $\lambda = 0.1$ reduced total travel distance by 1.58 km, or approximately 1.82%, and reduced pure travel time by 1.95 minutes compared with the hard time-window solution. However, these modest efficiency gains were accompanied by 8.95 minutes of total lateness across two customers. Both scenarios produced approximately the same total route exposure duration of 1,043.59 minutes and the same model-estimated fleet product loss of 507.95 kg. This finding indicates that shorter travel distance and pure travel time do not necessarily reduce melting-related product loss when the total duration of product exposure remains unchanged.

When the lateness penalty was increased to $\lambda = 0.5$ km/min, all customer lateness was eliminated and the soft time-window solution converged to the same customer allocation, route sequence, and 86.87-km fleet distance as the hard time-window solution. Increasing λ further to 1, 2, and 5 km/min produced no additional improvement. Therefore, the main result of the comparison is not that the soft time-window model universally outperforms the hard time-window model, but that the penalty parameter determines the trade-off between marginal routing efficiency and service-time compliance. In this case, $\lambda = 0.5$ km/min represents the minimum sufficient penalty required to obtain a zero-lateness solution under the modeled operational conditions.

Interpretation of Key Findings

The vehicle level differences can be explained by the interaction between route allocation, customer time windows, demand distribution, and road network structure. Under the hard time window solution, Truck 1 served 1,185 kg with 254 minutes of service time, whereas Truck 2 served 1,865 kg with 366 minutes of service time. Truck 2 therefore carried a larger

Table 8. Fleet Level Performance Comparison of Company Baseline and Optimized Routing Scenarios

Evaluation Criteria	Unit	Company Baseline	Soft-Constraints VRPTW ($\lambda = 0.1$)	Hard-Constraints VRPTW	Better Model
Total Lateness	minutes	Occurred (duration unavailable)	8.95	0.00	Hard-Constraints
Pure Travel Time	minutes	N/A	121.14	123.09	Soft-Constraints
Total Travel Distance	km	99.84	85.29	86.87	Soft-Constraints
Route Exposure Duration	minutes	~1200	~1,043.59	~1,043.59	Equal
Product Loss	kg	580 (observed)	507.95 (estimated)	507.95 (estimated)	Equal

Note: Customer lateness occurred under the company baseline; however, the exact lateness duration could not be quantified because the company did not maintain sufficiently detailed customer-level arrival and service-time records. Baseline pure travel time was also unavailable for the same reason.

share of the scheduled demand and served more customers over a longer route. At $\lambda = 0.1$ km/min, the soft solution changed not only the customer sequence but also the vehicle allocation. Most notably, customer C52 was reassigned from Truck 2 to Truck 1, while several early stops in the Truck 2 sequence were reordered. This reallocation produced different effects between vehicles: Truck 1 distance increased slightly from 30.99 to 31.22 km and pure travel time increased from 44.90 to 45.16 minutes, whereas Truck 2 distance decreased from 55.88 to 54.07 km and pure travel time decreased from 78.19 to 75.98 minutes. Thus, the fleet-level distance reduction of 1.58 km did not result from uniform improvement across both vehicles, but from a larger saving on Truck 2 that outweighed the small increase on Truck 1.

The different response of Truck 2 is also related to the temporal structure of its assigned customers. The early part of the hard Truck 2 route includes several customers with relatively tight morning service windows, including C43

(07:00–09:00), C19 (06:30–09:00), C13 (07:00–08:00), C11 (06:00–09:00), and C12 (07:30–08:30). Under the low-penalty soft scenario, the ordering of these early customers changed, allowing a reduction in road network distance but reducing the available temporal slack. This explains why Truck 2 obtained a larger distance saving but also generated the larger lateness value of 7.38 minutes. In contrast, Truck 1 experienced only a small distance increase and 1.57 minutes of lateness. These results indicate that soft time window performance depends on the spatial compatibility of customer assignments with the road network and on how much temporal flexibility remains within the assigned customer sequence, rather than on the soft-window formulation alone.

The penalty behavior further explains the transition between the $\lambda = 0.1$ and $\lambda = 0.5$ solutions. At $\lambda = 0.1$ km/min, the 8.95 minutes of fleet lateness corresponds to only 0.895 km of distance-equivalent penalty. Because the low-penalty soft route saves 1.58 km relative to the hard solution, the distance saving is larger than the associated lateness penalty



Figure 1. Optimized Hard Time-Window Route

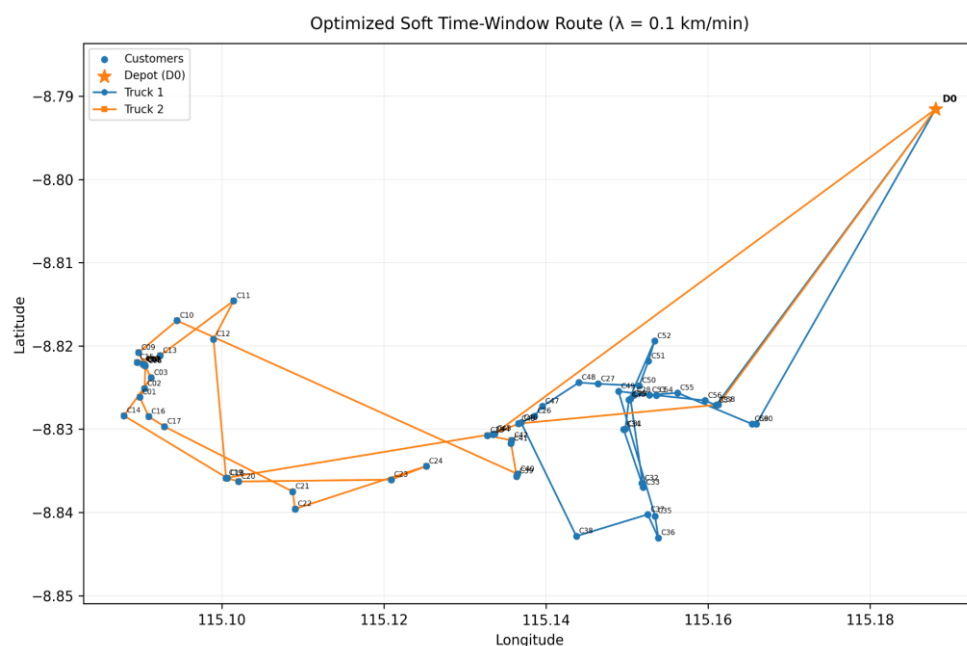


Figure 2. Optimized Soft Time-Window Route at $\lambda = 0.1$ km/min

and the algorithm can therefore favor the shorter but late solution. When λ is increased to 0.5 km/min, the same 8.95 minutes of lateness would correspond to a 4.475 km distance equivalent penalty, which is larger than the 1.58 km distance saving. Consequently, the lower-distance late solution is no longer attractive, and the soft model converges to the same 86.87 km zero lateness route as the hard model. The absence of further changes at $\lambda = 1, 2$, and 5 indicates that 0.5 km/min is sufficient to remove the economic incentive, within the generalized-distance objective, to exchange punctuality for the marginal distance reduction observed in this case.

Comparison with Previous Studies

The observed trade-off is consistent with the role of soft time windows described by (Figliozzi, 2010), who treated soft-window routing as a cost-service trade-off problem rather than requiring every time-window violation to make a route infeasible. Methodologically, however, Figliozzi used an iterative route-construction and improvement algorithm and evaluated its performance against established benchmark instances, reporting improvements in thirty benchmark solutions. The present study uses a simpler deterministic Cheapest Insertion Heuristic and does not claim comparable algorithmic advancement. Its contribution is instead to show, in an operational case, how the penalty coefficient changes the practical balance between a 1.82% fleet-distance saving at $\lambda = 0.1$ and complete time-window compliance at $\lambda = 0.5$.

The closest application context is the ice-distribution study of Thammasang & Arunyanart (2024). Their study used a more complex methodological framework, comparing an exact MILP model, a hierarchical cluster-first-route-second approach based on capacitated K-means clustering followed by MILP-based route optimization, and a Differential Evolution algorithm for larger instances. In their 83 customer real case, the proposed approach was reported to reduce the company's monthly transportation cost by approximately 23%. The magnitude should not be compared directly with the present study because the performance measures differ: Thammasang and Arunyanart evaluated total monetary cost, whereas the present case obtained a 12.99% reduction in fleet distance and an estimated 12.42% reduction in exposure-based product loss relative to the observed company baseline. The comparison nevertheless indicates that the operational benefit of routing optimization can be substantial in ice distribution, while the achievable effect depends strongly on the objective function, vehicle characteristics, customer structure, and solution method.

The present methodology also differs from more advanced perishable-product soft-window models such as Z. Liu et al. (2023), which considered electric vehicle routing, variable vehicle speed, refrigeration-related energy consumption, and soft time windows for perishable-product delivery and solved the resulting problem using an adaptive hybrid ant-colony approach. By comparison, the present study deliberately uses a simpler CIH-based framework with fixed non-refrigerated vehicles and static ORS road-network travel times. Therefore, the results should be interpreted as evidence that even a comparatively simple penalty-based heuristic can expose a meaningful distance-punctuality trade-off in a small-scale operational setting, rather than as evidence that the proposed procedure outperforms more advanced VRPTW algorithms.

Limitations and Cautions

Several limitations should be acknowledged. First, this study was conducted in a single crystal ice distribution company, so the findings may not be directly generalizable to other distribution systems with different customer patterns, fleet sizes, or vehicle conditions. Second, the product-loss estimation was based on a linear relationship between route

exposure duration and melting-related loss. Although this assumption is useful for practical estimation, actual melting may also be affected by temperature, vehicle insulation, loading conditions, and handling practices.

Third, this study used the Cheapest Insertion Heuristic, which is practical and computationally simple, but it does not guarantee a global optimal solution. Other algorithms, such as metaheuristics or exact optimization methods, may produce different routing results. Fourth, the travel-time matrix was based on estimated road-network travel time and may not fully capture real-time traffic variation, road congestion, or unexpected delivery delays.

Therefore, the results should be interpreted as practical route recommendations rather than absolute optimal solutions. The main contribution of this study lies in demonstrating how a simple penalty based soft time window approach can be applied to support decision-making in small-scale crystal ice distribution.

Recommendations for Future Research

Future research should prioritize three extensions. First, the routing procedure should be tested across additional perishable-distribution settings with different customer densities, fleet sizes, vehicle capacities, and delivery-area characteristics to assess the transferability of the observed distance-punctuality trade-off. Second, the product-loss model should be extended beyond the present linear exposure-based assumption by incorporating ambient temperature, vehicle-insulation performance, loading volume, and time-dependent thermal exposure. Such a formulation would allow melting-related deterioration to be represented more realistically throughout the delivery route. Third, the deterministic Cheapest Insertion Heuristic should be benchmarked against more advanced solution methods, particularly Adaptive Large Neighborhood Search, Genetic Algorithm, and exact optimization on tractable instances, using common routing and service-reliability indicators. These extensions would provide stronger evidence regarding both solution quality and the robustness of the proposed operational framework.

In practical terms, the company may use the hard time-window solution or the soft time-window model with $\lambda = 0.5$ km/min when complete customer time-window compliance is required, because both produced the same zero-lateness routing solution in this case. The lower penalty of $\lambda = 0.1$ km/min may provide a small reduction in travel distance, but this benefit is accompanied by measurable customer lateness. Therefore, the choice of penalty should consider the company's tolerance for late service and the operational importance of delivery punctuality rather than travel distance alone.

Conclusion

This study evaluated hard and penalty-based soft time-window routing for crystal-ice distribution using the Vehicle Routing Problem with Time Windows (VRPTW) and the Cheapest Insertion Heuristic. The hard time-window solution produced a fleet distance of 86.87 km with zero lateness. At $\lambda = 0.1$ km/min, the soft time-window model reduced the fleet distance to 85.29 km but generated 8.95 minutes of total lateness across two customers. Increasing λ to 0.5 km/min eliminated all lateness and produced the same 86.87-km route as the hard model. Therefore, $\lambda = 0.5$ km/min was identified as the minimum sufficient penalty within the tested range.

Compared with the company's baseline distribution condition, the selected zero-lateness solution reduced fleet distance from 99.84 to 86.87 km and route exposure duration from approximately 1,200 to 1,043.59 minutes. The exposure-based product-loss model estimated a reduction in fleet product loss from the observed baseline of 580 kg to 507.95 kg, corresponding to approximately 12.42%.

These findings indicate that soft time-window routing does not inherently outperform hard time-window routing. Instead, the penalty parameter determines the trade-off between marginal routing efficiency and service-time compliance. In this case, a low penalty provided a small distance advantage at the cost of lateness, whereas $\lambda = 0.5$ restored complete time-window compliance. From a conceptual perspective, the vehicle-level disaggregation further demonstrates that the effect of soft time-window flexibility may differ across individual routes even when fleet-level performance appears favorable, highlighting the importance of evaluating vehicle-specific route structure alongside aggregate fleet performance. The findings remain specific to the investigated small-scale, non-refrigerated crystal-ice distribution system and should be interpreted within the limitations of the case-study setting.

Author contributions

Rayyan Rafif Riansyah contributed to problem identification, data collection, model development, route optimization, data analysis, and manuscript drafting. Bryan Estavan Imanuel Sitanggang contributed to research supervision, methodological guidance, validation of the optimization model, and manuscript review. Mia Juliana contributed to conceptual guidance, interpretation of results, academic review, and final manuscript improvement. All authors read and approved the final manuscript.

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